Characteristic Impedance of Microstrip Lines

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Abstract **—It is shown that it is feasible to force the complex power P of a microstrip line to be given by the usual circuit definition:**

$$P = I^* V/2$$

where I and V are the current and voltage of the equivalent transmission line and * denotes complex conjugation. If this requirement is made, then the three common definitions of characteristic impedance (namely, the voltage–current, power–voltage, and power–current definitions) all become equivalent. The remaining arbitrariness in microstrip characteristic impedance Z_0 stems not from the choice of definition, as sometimes argued, but from the ability to choose one of the magnitudes $|I|$, $|V|$, and $|Z_0|$ for convenience, and also to choose the phase of either I or V (but not their relative phase). This clarification should make it easier to simplify equivalent circuits for drivers, loads, and discontinuities.

I. Introduction

FOR non-TEM structures such as microstrip lines it generally is agreed that the characteristic impedance is not unique. It also is agreed that the root of this uncertainty is the longitudinal field components of non-TEM modes. Because of these components, current and voltage are not uniquely defined in terms of the usual TEM path integrals of the fields. Faced with an ambiguity in the meaning of current I and voltage V a variety of results for the characteristic impedance Z_0 is obtained.

Usually, one of three definitions for Z_0 is used, as introduced by Schelkunoff [1], [2]:

1) the voltage–current definition:

$$Z_0 = V/I \tag{1}$$

2) the power–voltage definition:

$$Z_0 = |V|^2/(2P^*) \tag{2}$$

where P is the complex power, or

3) the power–current definition:

$$Z_0 = 2P/|I|^2 \tag{3}$$

where $*$ denotes complex conjugation, $|\ |$ denotes magnitude, and a time dependence $\exp(j\omega t)$ is assumed. Plainly, these definitions lead to a variety of results if I and V themselves are not unique. Examples can be found in Bianco *et al.* [3].

Does the choice of definition matter? For microstrip, authors usually state a preference for the power–curent definition, while for slotline they prefer a power–voltage definition. For example, Kuester, Chang, and Lewin [4] suggest "one may have to bear all three definitions in mind, and change between them as circumstances dictate." Jansen and Kirschning [5] state "the usefulness of any voltage per current definition appears to be restricted" and "what is required in practical [microstrip] design work... is obviously represented best by the power–current formulation." Jansen and Koster [6] go further, stating that the voltage–current definition "can be excluded *a priori*."

Such proposals are incomplete. Omitted is the very natural requirement upon any transmission line model that the complex power P satisfy the relation

$$P = I^* V/2. \tag{4}$$

If (4) is satisfied, then (2) and (3) follow from (1) by substitution of I or V in terms of P from (4). That is, as pointed out by Schelkunoff [2], all three definitions of characteristic impedance are identical if (4) applies.

If all three definitions of Z_0 should agree, then why is there controversy over which definition is the most suitable? Is there a reason not to adopt (4)? For example, Jansen and Koster [4] suggest that current and voltage might not provide an adequate description of a hybrid mode. Can the power condition (4) be satisfied in general? This note shows that (4) can be satisfied at least for the case where only one mode propagates, even if this is a hybrid mode. Therefore, all three definitions of Z_0 can be made to agree by using combinations of current and voltage compatible with the power given by (4).

II. Validity of the I^*V Relation for Power

The transmission line that is equivalent to a given microstrip line should propagate the same power. This power is given by the integral of Poynting's vector across the waveguide cross section, that is,

$$P = \tfrac{1}{2}\int dx \int dy \, \{ E_x H_y^* - E_y H_x^* \} \tag{5}$$

where propagation is in the z-direction, and E, H refer to the electric and magnetic fields. The power P from (5) is complex, and some authors restrict attention to only the real part of P, which corresponds to the time-averaged Poynting vector, and ignore the imaginary part, which corresponds to the reactive power circulated between the

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electric and magnetic fields [7]. Here, following Schelkunoff [2], both real and imaginary parts of the power are required to be the same in the microstrip line and its equivalent transmission line.

We now show that (5) can be made to agree with (4) for the case when only one mode can propagate (for example, the hybrid, quasi-TEM mode). The development parallels very closely the presentation of Marcuvitz [8, pp. 3–18] for waveguides. The major differences are as follows.

a) The dielectric permittivity is allowed to be complex and also a function of transverse position. That is, $\epsilon = \epsilon(x, y)$ where propagation is in the z-direction.

b) The magnetic permeability also is complex and $\mu = \mu(x, y)$.

As a result, the modes obtained from Maxwell's equations generally are hybrid modes, not the TE and TM modes used by Marcuvitz. The general functions $\epsilon(x, y)$ and $\mu(x, y)$ include lossy microstrip as a special case with piecewise constant ϵ and μ, and also other structures such as finlines and optical waveguides.

One finds even in this more general case that the fields can be represented as [9]

$$E(x, y, z) = E_t(x, y)V(z) + \eta E_l(x, y)I(z) \qquad (6)$$

$$H(x, y, z) = H_t(x, y)I(z) + \eta^{-1}H_l(x, y)V(z). \qquad (7)$$

Here the subscripts t and l refer to vectors transverse to and along the direction of propagation, and η is the intrinsic impedance of empty space in ohms, introduced only to make the dimensions of both the transverse and longitudinal components the same (inverse length). If the propagating mode were a TE or TM mode, then $E_t(x, y)$, $H_t(x, y)$ would become proportional to the mode functions $\{e', h'\}$ or $\{e'', h''\}$ of Marcuvitz [8, p. 4]. In (6) and (7), $V(z)$ is in V, $I(z)$ in A, $E(x, y, z)$ in V/m, $H(x, y, z)$ in A/m, and $E_t(x, y)$, $H_t(x, y)$ in m^{-1}.

As shown in [9], (6) and (7) satisfy Maxwell's equations provided that $I(z)$ and $V(z)$ satisfy the usual transmission line equations, namely,

$$\frac{dI(z)}{dz} = -\frac{\gamma}{Z_0}V(z) \qquad (8)$$

$$\frac{dV(z)}{dz} = -\gamma Z_0 I(z) \qquad (9)$$

which, of course, have the solutions

$$V(z) = A\exp(-\gamma z) + B\exp(\gamma z) \qquad (10)$$

$$I(z) = \frac{1}{Z_0}\{A\exp(-\gamma z) - B\exp(\gamma z)\}. \qquad (11)$$

From the solutions (10) and (11) it is evident that γ is the (complex) propagation constant of the mode. Its value is determined by the eigenvalue equation for the transverse mode amplitudes $E_t(x, y)$ or $H_t(x, y)$. However, there is no restriction on Z_0. That is, (6) and (7) are solutions of Maxwell's equations regardless of the value of Z_0. From (10) and (11), it is clear that the impedance defined by (1) is Z_0 only for the case of no reflected wave ($B = 0$), as also is true of (2) and (3).

Using (6) and (7) in (5), we find that (5) agrees with (4) provided that

$$\int dx \int dy \{E_{tx}H_{ty}^* - E_{ty}H_{tx}^*\} = 1. \qquad (12)$$

The left side of (12) is complex, while the right side is real. Therefore, it appears that (12) might be hard to satisfy. However, using Maxwell's equations and the transmission line equations (8) and (9), one finds that (12) can be reexpressed as [9]

$$\int dx \int dy \left\{\frac{1}{\mu}[\nabla \times E_t]\cdot[\nabla \times E_t]^* - \omega^2\epsilon|E_t|^2\right\} = j\omega\frac{\gamma}{Z_0} \qquad (13)$$

and

$$\int dx \int dy \left\{\frac{1}{\epsilon}[\nabla \times H_t]\cdot[\nabla \times H_t]^* - \omega^2\mu|H_t|^2\right\} = j\omega\gamma Z_0. \qquad (14)$$

Thus, the requirement that the complex power from Poynting's vector, (5), agree with the circuit definition, (4), is the same as the restrictions (13) and (14) upon the characteristic impedance Z_0. Using the customary definitions of the transmission line resistance R, inductance L, conductance G, and capacitance C (all per unit length), namely,

$$R + j\omega L = \gamma Z_0 \qquad (15)$$

$$G + j\omega C = \frac{\gamma}{Z_0} \qquad (16)$$

one finds that (13) and (14) are simple generalizations of common expressions for R, L, G, and C extended to the case where longitudinal field components (nonzero *curl* of the fields) exist [9]. Consequently, if Z_0 is chosen so that (4) applies to the complex power, the resulting R, L, G, and C reduce to the usual parameters [10] when the longitudinal field components are small.

It should be noted that E_t and H_t are connected by Maxwell's equations, so that (13) and (14) express the same condition on Z_0.[1] In addition, because the field (6) determines only the product of E_t and $V(z)$, E_t can be multiplied by an arbitrary complex constant, provided

[1] For example, from Maxwell's equations one finds

$$\nabla\left\{\frac{1}{\mu(x, y)}\nabla\cdot[\mu(x, y)H_t]\right\} + \gamma^2 H_t = j\omega\gamma Z_0\epsilon(x, y)(\hat{k} \times E_t)$$

and

$$\nabla\left\{\frac{1}{\epsilon(x, y)}\nabla\cdot[\epsilon(x, y)E_t]\right\} + \gamma^2 E_t = -j\omega\frac{\gamma}{Z_0}\mu(x, y)(\hat{k} \times H_t)$$

where $\hat{k}$ is a unit vector in the z-direction.

only that $V(z)$ is divided by the same constant. Consequently, only the phase of the left side of (13) or (14) is fixed. That is, the requirement that the two power expressions (4) and (5) be equal determines only the phase of the characteristic impedance, and its magnitude remains arbitrary.

At this point, one sees the natural requirement that the power be given by (4) can be satisfied, provided that the phase of the characteristic impedance is chosen to satisfy (13) and (14). Because satisfaction of (4) makes all the definitions (1)–(3) equivalent, there is no basic advantage to any particular choice of definition (1)–(3), and one's choice is decided by accessibility of I, V, or both.

III. Determination of Current and Voltage

Because of the longitudinal field components in a non-TEM structure, the usual line integrals of the field variables used to define current and voltage are path-dependent. This matter is discussed clearly by Getsinger [11]. However, more general definitions of current and voltage can be obtained from (6) and (7) using the condition (12). One finds [9]

$$I(z)=\left(\frac{1}{j\omega\gamma Z_0}\right)^*\int dx\int dy H(x,y,z)\cdot\left\{\nabla\times\left[\frac{1}{\epsilon}\nabla\times H_t\right]-\omega^2\mu H_t\right\}^* \quad (17)$$

and

$$V(z)=\left(\frac{Z_0}{j\omega\gamma}\right)^*\int dx\int dy E(x,y,z)\cdot\left\{\nabla\times\left[\frac{1}{\mu}\nabla\times E_t\right]-\omega^2\epsilon E_t\right\}^*. \quad (18)$$

Equations (17) and (18) show that "current" and "voltage" are abstract entities, related to weighted averages of the transverse fields across the waveguide cross section (cf. Marcuvitz [8], p. 5). The definitions (17) and (18) do not involve path integrals, and are well defined even when longitudinal field components are present. However, (17) and (18) do not determine a unique I or V until the magnitude and phase of H_t or E_t are decided.

For example, if the magnitude of Z_0 is fixed, then the magnitude of E_t is determined by (13) and that of H_t is determined by (14). These two equations also determine the phase of Z_0, as already discussed. With the magnitudes of the transverse modes determined, (17) and (18) determine the current and voltage, with phases determined by the phases of H_t and E_t, respectively.[2]

[2]Although the phases of I and V are determined by (17) and (18), their relative phase is not arbitrary. That is, the phase of either H_t or E_t can be chosen arbitrarily, but their relative phase is fixed by Maxwell's equations (see Footnote 1). Thus, the relative phase of I and V also is not arbitrary, in accordance with (1).

However, the formulation based upon (13), (14), (17), and (18) is not restricted to choosing the magnitude of Z_0 first, and then determining I and V. If it is more convenient, simple rearrangement of the equations allows voltage (or current) to be chosen first, with current (or voltage) and $|Z_0|$ determined afterwards. For example, (14) can be used to eliminate Z_0 from (17), thereby determining the magnitude and phase of H_t once $I(z)$ is given. Then Z_0 can be obtained from (14), E_t from H_t, and $V(z)$ from (18). This procedure serves to generalize (1)–(3) because it works even when reflected waves are present.

As a result of this interdependence of I and V, it is not surprising that some simple choices for current and voltage are not compatible. For example, if one takes current as total longitudinal strip current in definition (3) and voltage as strip center-voltage in definition (2) without regard for compatibility, then it is not surprising that the different definitions produce different Z_0's.

IV. The Role of Convenience

Although many authors use a current or a voltage variable chosen in an arbitrary manner, this choice often is considered as merely expedient, as of uncertain status, or as tied to the choice of one of the definitions (1)–(3). In fact, such choice is legitimate and unrestricted: one is free to select from I, V, and $|Z_0|$ whichever variable is more convenient, and to ascribe an arbitrary amplitude to the variable of choice.

For example, suppose one elects current as the independent variable. According to (7), the component of magnetic field normal to the direction of propagation, $H_n(x,y,z)$, satisfies

$$I(z)=H_n(x,y,z)/H_t(x,y). \quad (19)$$

That is, *if current is selected as the independent variable*, the only requirement upon $I(z)$ is that its z-dependence be proportional to that of the normal component of the magnetic field of the propagating mode. Otherwise, (19) could not be satisfied. The constant of proportionality is arbitrary and can vary with frequency because it determines only the arbitrary normalization of $H_t(x,y)$. Once $I(z)$ is selected, thereby fixing the magnitude and phase of $H_t(x,y)$, Z_0 is fixed by (14), and $E_t(x,y)$ is fixed by Maxwell's equations (see Footnote 1). Then $E_t(x,y)$ in turn determines $V(z)$ through (6) or (18).

In the same way, if one selects voltage as the independent variable, then from (6) or (18) the component of electric field normal to the direction of propagation, $E_n(x,y,z)$, satisfies

$$V(z)=E_n(x,y,z)/E_t(x,y). \quad (20)$$

That is, *if voltage is selected as the independent variable*, the only requirement upon $V(z)$ is that its z-dependence be proportional to the z-dependence of the normal component of the electric field of the propagating mode. Again, the constant of proportionality is arbitrary, corresponding to a choice of magnitude and phase for $E_t(x, y)$.

One can exploit this arbitrariness. For example, Kuester *et al.* [4] have shown that the analysis of a slot voltage driver in a microstrip line is formulated naturally in terms of the longitudinal strip current. This current satisfies (19), so it is convenient and perfectly general to choose the abstract transmission line current to be this strip current. This choice does not restrict one to the use of a "power–current" definition of impedance. However, it does determine a corresponding voltage and, if this voltage is used, all the definitions (1)–(3) will produce the same impedance as the "power–current" definition for the case of no reflected waves.

It also may happen that one wishes to use a simple TEM equivalent circuit for the microstrip line. So long as the circuit has the correct phase for Z_0, one can choose Z_0 to be the TEM impedance. However, then the current and voltage are determined to within a common phase factor, and need not be the TEM current and voltage. Satisfaction of Kirchhoff's laws at the junction between the transmission line and any load, driver, or discontinuity is arranged by adjustment of the equivalent circuit that represents this junction.

In short, choosing current and voltage for convenience is subject to remarks similar to those for waveguides made earlier by Marcuvitz [8, pp. 8, 105, 119]. In particular, he says, "Although the voltage, V, and current, I, suffice to characterize the behavior of a mode, it is evident that such a characterization is not unique. Occasionally it is desirable to redefine the relations between the fields and the voltage and current in order to correspond more closely to customary low-frequency definitions, or to simplify the equivalent circuit description of waveguide discontinuities. These redefinitions introduce changes of the form [generalized here to complex N]

$$V = \overline{V}/(N)^{1/2} \qquad I = (N^*)^{1/2}\overline{I} \tag{21}$$

where the scale factor N is so chosen as to retain the form of the power expression." Using this transformation, the impedance becomes

$$Z_0 = \overline{Z}_0/|N|. \tag{22}$$

Note that (22) does not alter the phase of Z_0, and (21) does not alter the relative phase of I and V.

V. Controversy Revisited

Once (4) is adopted, controversy over the choice of definition (1)–(3) is ended. However, one still may discuss the practical value of different impedances for different problems, and much of this discussion in the existing literature remains valid with a shift in the definitions of "power-current" and "power-voltage" impedances. Instead of taking these terms to indicate an equation of definition (which is of no consequence), one may take them to indicate that the "power" is given by (4) and that the "current" or the "voltage" is the variable chosen for convenience. Corresponding to a variety of quantities that can represent the "current" (or "voltage"), there is a variety of "power–current" (or "power–voltage") impedances.

The term "voltage–current" impedance is harder to rehabilitate. To be consistent with (4), this term should mean that the magnitude of the ratio of voltage to current (that is, $|Z_0|$) was chosen as the variable of convenience. However, in the literature, the "voltage–current" definition usually combines a voltage and a current incompatible with (4). This incompatibility probably causes the behavior leading to the negative comments about "voltage–current" impedances quoted in the Introduction.

VI. Summary

It has been shown that it is feasible to impose the usual current–voltage relation for complex power (4) for the case when one mode propagates, even if this mode is a hybrid mode of a non-TEM structure. Once imposed, the complex power condition (4) forces all the definitions of characteristic impedance (1)–(3) to agree. With this complex power condition imposed, current, voltage, and magnitude of the characteristic impedance all are interdependent. One is free to choose only one of the magnitudes $|I|$, $|V|$, and $|Z_0|$ arbitrarily and also to choose the phase of either I or V (but not their relative phase).

As the requirement (4) for complex power agrees with ordinary circuit theory, and certainly is useful in applying any equivalent circuit, it seems reasonable to adopt it. Although such adoption does not lead to a unique characteristic impedance, it ends any debate over the choice of definitions (1)–(3). Instead, it places attention on the simplification of circuits through the choice of current, voltage, or magnitude of characteristic impedance.

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